IJMRSETM

e-ISSN: 2395 - 7639

INTERNATIONAL JOURNAL OF MULTIDISCIPLINARY RESEARCH

IN SCIENCE, ENGINEERING, TECHNOLOGY AND MANAGEMENT

Volume 10, Issue 5, May 2023

INTERNATIONAL STANDARD SERIAL NUMBER INDIA

Impact Factor: 7.580

+91 99405 72462 +9163819 07438 ijmrsetm@gmail.com www.ijmrsetm.com

| ISSN: 2395-7639 | www.ijmrsetm.com | Impact Factor: 7.580| A Monthly Double-Blind Peer Reviewed Journal |

| Volume 10, Issue 5, May 2023 |

The Implication of Friedmann–Lemaître–Robertson–Walker metric (FLRW) in Relativistic Cosmology

Pallavi Saikia

Former Assistant Professor, Department of Mathematics, Kokrajhar Government College, Assam, India

ABSTRACT: The appearance of objects at cosmological distances is affected by thecurvature of spacetime through which light travels on its way to Earth. Themost complete description of the geometrical properties of the Universe isprovided by Einstein's general theory of relativity. In General Relativity, the fundamentalquantity is the metric which describes the geometry of spacetime.

KEYWORDS: cosmology, relativity, metric, geometry, Earth.

I. INTRODUCTION

Friedmann Robertson Walker Cosmological Model:

The **Friedmann–Lemaître–Robertson–Walker metric** (**FLRW**) is a metric based on the exact solution of the Einstein's field equations of general relativity. The metric describes a homogeneous, isotropic, expanding (or otherwise, contracting) universe that is path-connected, but not necessarily simply connected. The general form of the metric follows from the geometric properties of homogeneity and isotropy; Einstein's field equations are only needed to derive the scale factor of the universe as a function of time.

As Metric is nothing but a distance function, there are three different types of distances:

- Hamming distance calculates the distance between two binary vectors, also referred to as binary strings or bitstrings for short.
- Euclidean distance calculates the distance between two real-valued vectors.
- Minkowski distance calculates the distance between two real-valued vectors.

General Models of the Universe:

There are two general models. Heliocentric and Geocentric are two explanations of the arrangement of the universe, including the solar system.

General metric

The FLRW metric starts with the assumption of homogeneity and isotropy of space. It also assumes that the spatial component of the metric can be time-dependent. The generic metric which meets these conditions is

$$-c^2 d\boldsymbol{\tau}^2 = -\mathbf{c}^2\mathbf{dt}^2 + a(t)^2 d\boldsymbol{\Sigma}^2$$

where Σ ranges over a 3-dimensional space of uniform curvature, that is, elliptical space, Euclidean space, or hyperbolic space. It is normally written as a function of three spatial coordinates, but there are several conventions for doing so.

II. REVIEW OF THE RELATED LITERATURE

Depending on geographical or historical preferences, the set of the four scientists – Alexander Friedmann, Georges Lemaître, Howard P. Robertson and Arthur Geoffrey Walker – are variously grouped as Friedmann, Friedmann–Robertson–Walker (FRW), Robertson–Walker (RW), or Friedmann–Lemaître (FL). This model is sometimes called the Standard Model of modern cosmology, although such a description is also associated with the further developed Lambda-CDM model. The FLRW model was developed independently by the named authors in the 1920s and 1930s.

The Soviet mathematician Alexander Friedmann first derived the main results of the FLRW model in 1922 and 1924.[12][13] Although the prestigious physics journal Zeitschrift für Physik published his work, it remained relatively unnoticed by his contemporaries. Friedmann was in direct communication with Albert Einstein, who, on behalf of Zeitschrift für Physik, acted as the scientific referee of Friedmann's work. Eventually Einstein acknowledged the correctness of Friedmann's calculations, but failed to appreciate the physical significance of Friedmann's predictions.

Friedmann died in 1925. In 1927, Georges Lemaître, a Belgian priest, astronomer and periodic professor of physics at the Catholic University of Leuven, arrived independently at results similar to those of Friedmann and published them in the Annales de la Société Scientifique de Bruxelles (Annals of the Scientific Society of Brussels).[14][15] In the face of the observational evidence for the expansion of the universe obtained by Edwin Hubble in the late 1920s, Lemaître's results were noticed in particular by Arthur Eddington, and in 1930–31 Lemaître's paper was translated into English and published in the Monthly Notices of the Royal Astronomical Society.

Howard P. Robertson from the US and Arthur Geoffrey Walker from the UK explored the problem further during the 1930s. In 1935 Robertson and Walker rigorously proved that the FLRW metric is the only one on a spacetime that is spatially homogeneous and isotropic (as noted above, this is a geometric result and is not tied specifically to the equations of general relativity, which were always assumed by Friedmann and Lemaître).

This solution, often called the Robertson–Walker metric since they proved its generic properties, is different from the dynamical "Friedmann–Lemaître" models, which are specific solutions for a(t) which assume that the only contributions to stress–energy are cold matter ("dust"), radiation, and a cosmological constant.

The Friedmann–Lemaître–Robertson–Walker models (often FRW-models) are class of models in cosmology. These are solutions to Einstein's equations describing a spatially homogeneous and isotropic expanding or contracting spacetime. Hence these are solutions used as models in cosmology.

Robertson-Walker Metric Geodesic

The Robertson-Walker metric gives mathematical expression to three widely-held assumptions about the nature of the observable universe. It is seen that the null-geodesic of this metric has little-known solutions for the speed and distance of a light-signal relative to its source.

Current status

The current standard model of cosmology, the Lambda-CDM model, uses the FLRW metric. By combining the observation data from some experiments such as WMAP and Planck with theoretical results of Ehlers–Geren–Sachs theorem and its generalization,[24] astrophysicists now agree that the early universe is almost homogeneous and isotropic (when averaged over a very large scale) and thus nearly a FLRW spacetime. That being said, attempts to confirm the purely kinematic interpretation of the Cosmic Microwave Background (CMB) dipole through studies of radio galaxies and quasars show disagreement in the magnitude. Taken at face value, these observations are at odds with the Universe being described by the FLRW metric. Moreover, one can argue that there is a maximum value to the Hubble constant within an FLRW cosmology tolerated by current observations and depending on how local determinations converge, this may point to a breakdown of the FLRW metric in the late universe, demanding an explanation beyond the FLRW metric.

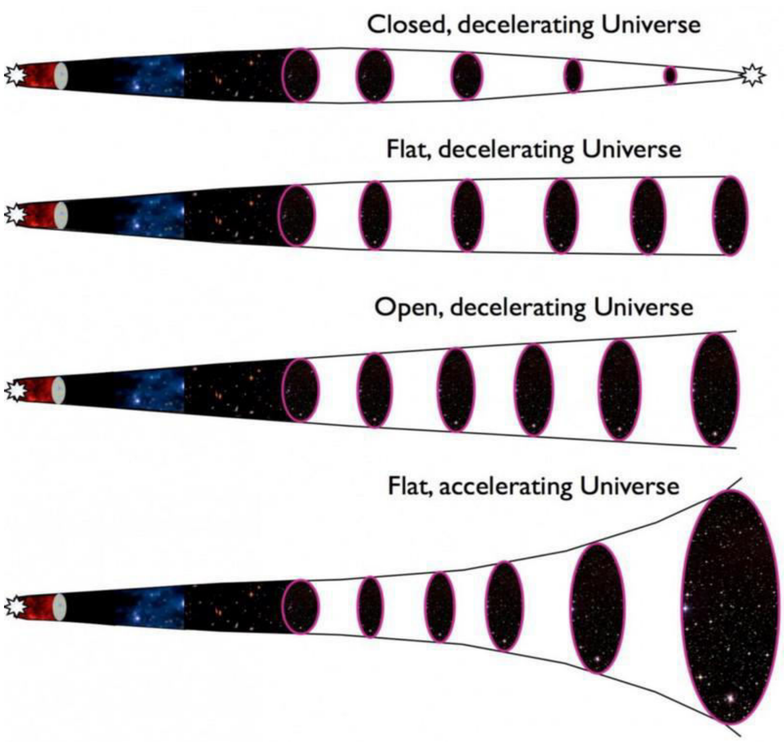
The Most Important Equation in The Universe

The first Friedmann equation describes how, based on what is in the universe, its expansion rate will change over time. If you want to know where the Universe came from and where it's headed, all you need to measure is how it is expanding today and what is in it. The story of Friedmann, his equation, and what it teaches us about our Universe is a story that every science enthusiast should know.

In 1915, Einstein put forth his theory of General Relativity, which related the curvature of spacetime on one hand to the presence of matter and energy in the Universe on the other. As John Wheeler put it many years later, spacetime tells matter how to move; matter tells spacetime how to curve. Einstein's theory, in one fell swoop, reproduced all the previous successes of Newton's gravity, explained the intricacies of Mercury's orbit (which Newton's theory couldn't), and made a new prediction for the bending of starlight, which was spectacularly confirmed during the total solar eclipse of 1919. The only problem? In order to prevent the Universe from collapsing in on itself, Einstein needed to add a cosmological constant — an ad hoc fix for the fact that static spacetimes were unstable in General Relativity — to his theory. It was ugly, it was finely-tuned, and it had no other motivation.

Alexander Friedmann was just 33 when he wrote down the Friedmann equations and predictions.

In 1922, just three years after the eclipse confirmation, Friedmann found an elegant way to save the Universe while simultaneously doing away with the cosmological constant: don't assume that it's static. Instead, Friedmann argued, assume that it is as we observe it, full of matter and radiation, and allowed to be curved. Assume, further, that it's roughly isotropic and homogeneous, which are mathematical words meaning "the same in all directions" and "the same at all locations." If you make these assumptions, two equations pop out: the Friedmann equations. They tell you that the Universe isn't static, but rather that it either expands or contracts depending on what the expansion rate and the contents of your Universe are. Best of all, they tell you how the Universe evolves with time, arbitrarily far into the future or past.



What's remarkable is that Friedmann put this out before we discovered that the Universe was expanding; before Hubble even discovered that there were galaxies beyond the Milky Way in the Universe! It wouldn't be until the next year that Hubble would identify Cepheid variable stars in Andromeda, teaching us its distance and placing it far outside of our own galaxy. Furthermore, it wouldn't be until the late 1920s that Georges Lemaître and later, independently, Hubble, would put the redshift-and-distance figures together to conclude that the Universe was expanding.

Yet Hubble's scientific legacy was indisputable, and became even more so as we came to understand cosmology better. The first Friedmann equation is the most important of the two, since it's the most easy and straightforward to tie to observations. On one side, you have the equivalent of the expansion rate (squared), or what's colloquially known as the Hubble constant. (It's not truly a constant, since it can change as the Universe expands or contracts over time.) It tells you how the fabric of the Universe expands or contracts as a function of time.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

The first Friedmann equation, as conventionally written today (in modern notation), where the left

On the other side is literally everything else. There's all the matter, radiation, and any other forms of energy that make up the Universe. There's the curvature intrinsic to space itself, dependent on whether the Universe is closed (positively curved), open (negatively curved), or flat (uncurved). And there is also the Λ term: a cosmological constant, which can either be a form of energy or can be an intrinsic property of space.

Either way, this is the equation that relates how the Universe expands, quantitatively, to what makes up the matter and energy within it. Measure what's in your Universe today and how fast it's expanding today, and you can extrapolate forwards or backwards by arbitrary amounts. You can know how the Universe was expanding in the distant past or immediately after the Big Bang. You can know whether it will recollapse or not (it won't), or whether the expansion rate will asymptote to zero (it won't) or remain positive forever (it will).

And perhaps most spectacularly, you can add imperfections atop this smooth background. The density imperfections you put into your Universe tell you how large-scale structure grows and forms, what will grow into a galaxy/cluster and what won't, and what will become gravitationally bound versus what will be driven apart.

All of this can be derived from one single equation: the first Friedmann equation.

There is a large suite of scientific evidence that supports the picture of the expanding Universe

Although Friedmann's life was short, his influence cannot be overstated. He was the first to derive the General Relativity solution that describes our Universe: an expanding Universe filled with matter. Although it was independently derived, later, by three others — Georges Lemaître, Howard Robertson, and Arthur Walker — Friedmann fully realized its implications and applications, and even came up with the first solutions for exotically curved spaces. He was an influential teacher as well; his most famous pupil was George Gamow, who would later go on to apply Friedmann's work to the expanding Universe to create the Big Bang Theory of our cosmic origin.

Nearly a century after his most famous work, Friedmann's equations have been extended to a Universe containing an inflationary origin, dark matter, neutrinos, and dark energy. Yet they're still perfectly valid, with no additions or modifications required to account for these tremendous advances. While we can all argue about the relative merits of Einstein, Newton, Maxwell, Feynman, Boltzmann, Hawking, and many others, when it comes to the expanding Universe, Friedmann's first equation is the only one you need. It connects the matter and energy that's present to the expansion rate today, in the past, and in the future, and allows you to know the fate and history of the Universe from measurements we can make today. As far as the fabric of our Universe is concerned, this equation takes the crown as the single most important.

III. CONCLUSION

Unsolved problem in physics

Is the universe homogeneous and isotropic at large enough scales, as claimed by the cosmological principle and assumed by all models that use the Friedmann–Lemaître–Robertson–Walker metric, including the current version of ΛCDM, or is the universe inhomogeneous or anisotropic? Is the CMB dipole purely kinematic, or does it signal a possible breakdown of the FLRW metric?Even if the cosmological principle is correct, is the Friedmann–Lemaître–Robertson–Walker metric valid in the late universe?

REFERENCES

1. For an early reference, see Robertson (1935); Robertson assumes multiple connectedness in the positive curvature case and says that "we are still free to restore" simple connectedness.
2. M. Lachieze-Rey; J.-P. Luminet (1995), "Cosmic Topology", Physics Reports, **254** (3): 135–214, arXiv:gr-qc/9605010, Bibcode:1995PhR...254..135L, doi:10.1016/0370-1573(94)00085-H, S2CID 119500217
3. G. F. R. Ellis; H. van Elst (1999). "Cosmological models (Cargèse lectures 1998)". In Marc Lachièze-Rey (ed.). Theoretical and Observational Cosmology. NATO Science Series C. Vol. 541. pp. 1–116. arXiv:gr-qc/9812046. Bibcode:1999ASIC..541....1E. ISBN 978-0792359463.
4. L. Bergström, A. Goobar (2006), Cosmology and Particle Astrophysics (2nd ed.), Sprint, p. 61,ISBN 978-3-540-32924-4
5. Wald, Robert. General Relativity. p. 116.
6. Carroll, Sean. Spacetime and Geometry: An Introduction to General Relativity. pp. 329–333.
7. Wald, Robert. General Relativity. p. 97.
8. "Cosmology"(PDF). p. 23.
9. P. Ojeda and H. Rosu (2006), "Supersymmetry of FRW barotropic cosmologies", International Journal of Theoretical Physics, **45** (6): 1191–1196, arXiv:gr-qc/0510004, Bibcode:2006IJTP...45.1152R, doi:10.1007/s10773-006-9123-2, S2CID 119496918
10. Their solutions can be found in Rosu, Haret C.; Mancas, S. C.; Chen, Pisin (2015-05-05). "Barotropic FRW cosmologies with Chiellini damping in comoving time". Modern Physics Letters A. **30** (20): 1550100. arXiv:1502.07033. Bibcode:2015MPLA...3050100R. doi:10.1142/S021773231550100x. ISSN 0217-7323. S2CID 51948117.
11. McCrea, W. H.; Milne, E. A. (1934). "Newtonian universes and the curvature of space". Quarterly Journal of Mathematics. **5**: 73–80. Bibcode:1934QJMat...5...73M. doi:10.1093/qmath/os-5.1.73.
12. Friedmann, Alexander (1922), "Über die Krümmung des Raumes", Zeitschrift für Physik A, **10** (1): 377–386, Bibcode:1922ZPhy...10..377F, doi:10.1007/BF01332580, S2CID 125190902
13. Friedmann, Alexander (1924), "Über die Möglichkeiteiner Welt mitkonstanternegativerKrümmung des Raumes", Zeitschrift für Physik A, **21** (1): 326–332, Bibcode:1924ZPhy...21..326F, doi:10.1007/BF01328280, S2CID 120551579 English trans. in 'General Relativity and Gravitation' 1999 vol.31, 31–
14. Lemaître, Georges (1931), "Expansion of the universe, A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulæ", Monthly Notices of the Royal Astronomical Society, **91** (5): 483–490, Bibcode:1931MNRAS..91..483L, doi:10.1093/mnras/91.5.483 translated from Lemaître, Georges (1927), "Un univershomogène de masse constante et de rayon croissant rendantcompte de la vitesse radiale des nébuleuses extra-galactiques", Annales de la Société Scientifique de Bruxelles, **A47**: 49–56, Bibcode:1927ASSB...47...49L
15. Lemaître, Georges (1933), "l'Universen expansion", Annales de la Société Scientifique de Bruxelles, **A53**: 51–85, Bibcode:1933ASSB...53...51L
16. Robertson, H. P. (1935), "Kinematics and world structure", Astrophysical Journal, **82**: 284–301, Bibcode:1935ApJ....82..284R, doi:10.1086/143681
17. Robertson, H. P. (1936), "Kinematics and world structure II", Astrophysical Journal, **83**: 187–201, Bibcode:1936ApJ....83..187R, doi:10.1086/143716
18. Robertson, H. P. (1936), "Kinematics and world structure III", Astrophysical Journal, **83**: 257–271, Bibcode:1936ApJ....83..257R, doi:10.1086/143726
19. Walker, A. G. (1937), "On Milne's theory of world-structure", Proceedings of the London Mathematical Society, Series 2, **42** (1): 90–127, Bibcode:1937PLMS...42...90W, doi:10.1112/plms/s2-42.1.90
20. Elcio Abdalla; Guillermo Franco Abellán; et al. (11 Mar 2022), "Cosmology Intertwined: A Review of the Particle Physics, Astrophysics, and Cosmology Associated with the Cosmological Tensions and Anomalies", Journal of High Energy Astrophysics, **34**: 49, arXiv:2203.06142v1, Bibcode:2022JHEAp..34...49A, doi:10.1016/j.jheap.2022.04.002, S2CID 247411131
21. Lee Billings (April 15, 2020). "Do We Live in a Lopsided Universe?". Scientific American. Retrieved March 24, 2022.
22. Migkas, K.; Schellenberger, G.; Reiprich, T. H.; Pacaud, F.; Ramos-Ceja, M. E.; Lovisari, L. (8 April 2020). "Probing cosmic isotropy with a new X-ray galaxy cluster sample through the LX-T scaling relation". Astronomy & Astrophysics. **636** (April 2020): 42. arXiv:2004.03305. Bibcode:2020A&A...636A..15M. doi:10.1051/0004-6361/201936602. S2CID 215238834. Retrieved 24 March 2022.
23. Krishnan, Chethan; Mohayaee, Roya; Colgáin, Eoin Ó; Sheikh-Jabbari, M. M.; Yin, Lu (16 September 2021). "Does Hubble Tension Signal a Breakdown in FLRW Cosmology?". Classical and Quantum Gravity. **38** (18):

184001. arXiv:2105.09790. Bibcode:2021CQGra..38r4001K. doi:10.1088/1361-6382/ac1a81. ISSN 0264-9381. S2CID 234790314.

24. Secrest, Nathan J.; Hausegger, Sebastian von; Rameez, Mohamed; Mohayaee, Roya; Sarkar, Subir; Colin, Jacques (2021-02-25). "A Test of the Cosmological Principle with Quasars". The Astrophysical Journal. **908** (2): L51. arXiv:2009.14826. Bibcode:2021ApJ...908L..51S. doi:10.3847/2041-8213/abdd40. S2CID 222066749.
25. Krishnan, Chethan; Mohayaee, Roya; Ó Colgáin, Eoin; Sheikh-Jabbari, M. M.; Yin, Lu (2021-05-25). "Does Hubble tension signal a breakdown in FLRW cosmology?". Classical and Quantum Gravity. **38** (18): 184001. arXiv:2105.09790. Bibcode:2021CQGra..38r4001K. doi:10.1088/1361-6382/ac1a81. S2CID 234790314.